(21)

This question paper contains 2 printed pages.

Your Roll No. 5/12/17

Sl. No. of Ques. Paper: 5706 **H**

Unique Paper Code : **235101**

Name of Paper : **Calculus – I**

Name of Course : **B.Sc. (Hons.) Mathematics**

Semester : **I**

Duration : **3 hours**

Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All Sections are compulsory. All questions carry equal marks. Use of non-programmable scientific calculator is allowed.

SECTION – I

Attempt any ***four*** questions from Section – I

1. If $y = \cos(m\sin^{-1}x)$, show that
$$(1-x^2)\,y_{n+2} - (2n+1)\,x\,y_{n+1} - (m^2 - n^2)\,y_n = 0.$$

2. Show that in general the graph of function
$$f(x) = \frac{ax^2+bx+c}{rx^2+sx+t}$$
has $y = \frac{a}{r}$ as a horizontal asymptote and that when $br \neq as$, the graph will cross this asymptote at the point where $x = \frac{at-cr}{br-as}$.

3. Find all values of A and B such that $\lim_{x\to 0} \frac{\sin Ax + Bx}{x^3} = 36$.

4. The total cost (in dollars) of manufacturing x units of a commodity is
$$C(x) = 3x^2 + 5x + 75$$
   (i) At what level of production is the average cost per unit the smallest?
   (ii) At what level of production is the average cost per unit equals the marginal cost?

5. Sketch the curve $r = 2\cos 3\theta$ in polar coordinates.

P. T. O.

Section –II

Attempt any ***four*** questions from Section II.

6. Evaluate: $\int \sin^4 x \cos^4 x\,dx$

7. Find the arc length of the curve $y = 3x^{3/2} - 1$ from $x=0$ to $x=1$.

8. Derive the formula for the volume of a sphere of radius r.

9. Find the area of the surface that is generated by revolving the portion of the curve $x = y^3$ between $y=0$ and $y=1$ about y-axis.

10. Find the volume of the solid generated when the region between the curves $y = \sin x$, $y = \cos x$, $x = 0$, $x = \pi/4$ is revolved about x-axis.

Section III

Attempt any ***three*** questions from Section III.

11. Describe the graph of the equation $9x^2 + 4y^2 + 18x - 24y + 9 = 0$

12. Find an equation for the ellipse with foci (1, 2) and (1, 4) and minor axis of length 2.

13. Show that graph of the equation

$$\sqrt{x} + \sqrt{y} = 1$$

is a portion of a parabola.

14. Find a polar equation for the ellipse that has its focus at the pole, directrix to the right of the pole; $a = 8$; $e = \frac{1}{2}$.

Section IV

Attempt any ***four*** questions from Section IV

15. Find the tangent vector to the graph of the vector function

$$\overrightarrow{F(t)} = e^{2t}\hat{i} + (t^2 - t)\hat{j} + (\ln t)\hat{k}\text{; at the points:}$$

$$t = 0.2,\ t = 0.5,\ t = 1.$$

16. Find the position vector $\overline{R(t)}$ of a moving particle if its acceleration vector is

$$\overline{A(t)} = t^2\hat{i} - 2\sqrt{t}\,\hat{j} + e^{3t}\hat{k}$$

initial position $\overline{R(0)} = \hat{i} + 2\hat{j}$ and initial velocity $\overline{V(0)} = 0$.

17. A boy standing at the edge of a cliff throws a ball upward at an angle of 30^0 with the horizontal axis and an initial speed of 64 ft/ s. Suppose that when the ball leaves the boy's hand, it is 48 ft. above the ground at the base of the cliff. What are the time of flight of the ball and its range?

18. The velocity $\overline{V_0} = 5\hat{i} - \hat{j} + 2\hat{k}$ and the acceleration $\overline{A_0} = \hat{i} - 7\hat{k}$ of a moving object are given. Find the normal and the tangential component of acceleration of the object at that instant.

19. The position vector of a moving body is $\overline{R(t)} = 2t\,\hat{i} - t^2\hat{j}$ for $t \geq 0$. Express R and the velocity vector $\overline{V(t)}$ in terms of unit polar vectors $\hat{u}_r$ and $\hat{u}_\theta$.

99

This question paper contains 2 printed pages.

Your Roll No. 7/12/17

Sl. No. of Ques. Paper: **5707** **H**

Unique Paper Code : **235103**

Name of Paper : **Analysis – I (MAHT-102)**

Name of Course : **B.Sc. (Hons.) Mathematics**

Semester : **I**

Duration : **3 hours**

Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory and carry equal marks. Attempt any three parts from each question.

Q1 (a) If $x > -1$, then prove that $(1 + x)^n \geq 1 + nx, \forall n \in \mathbb{N}$.

(b) Prove that there does not exist a rational number r such that $r^2 = 3$.

(c) Consider the function f defined by

$$f(x) = \frac{2x^2 + 3x + 1}{2x - 1} \; for \; 2 \leq x \leq 3.$$

Find a constant M such that $|f(x)| \leq M. \forall \; 2 \leq x \leq 3$.

(d) Define infimum of a set. Prove that a lower bound l of a non-empty set S in $\mathbb{R}$ is the infimum of S if and only if for every $\epsilon > 0$, there exists an $s_\epsilon \in S$ such that $l + \epsilon > s_\epsilon$.

Q 2 (a) Prove that there exists a positive real number x such that $x^2 = 2$.

(b) Define cluster point of a set. Also, prove that a subset of $\mathbb{R}$ is closed if and only if it contains all of its cluster points.

(c) Using the definition of limit of a sequence, show that $\lim_{n\to\infty}\left(\frac{3n+2}{n+1}\right) = 3$.

(d) Define a convergent sequence. Show that every convergent sequence is bounded. Give an example to show that converse is not true.

Q 3 (a) If $0 < b < 1$, then show that $\lim_{n\to\infty} b^n = 0$.

(b) State and prove that Monotone Convergence Theorem.

P. T. O.

(c) Let $X = (x_n)$ be a sequence of real numbers that converges to x and suppose that $x_n \geq 0$. Then, prove that the sequence $(\sqrt{x_n})$ of positive square roots converges and $\lim_{n\to\infty}(\sqrt{x_n}) = \sqrt{x}$.

(d) Let (x_n) be a sequence of real numbers such that $x_1 = 1$ and $x_{n+1} = \sqrt{2x_n}$, for $n \in \mathbb{N}$. Show that the sequence (x_n) is convergent. Also, find $\lim_{n\to\infty}(x_n)$.

Q 4 (a) Prove that a sequence of real numbers is Cauchy if and only if it is convergent.

(b) Show that the sequence (x_n) given by

$$x_n = \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}$$

is convergent.

(c) Prove that

$$\sum_{n=0}^{\infty} a\, r^n = \frac{a}{1-r}, \qquad if\ |r| < 1.$$

(d) Check whether the following series converge or diverge.

$$(i) \quad \sum_{n=0}^{\infty} \frac{n}{n^2+3}$$

$$(ii) \quad \sum_{n=0}^{\infty} \frac{n}{3^n}$$

Q 5 (a) State ratio test for the series of real numbers. Also, give examples in which test applies and test fails.

(b) Show that the series $\sum_{n=0}^{\infty} \frac{1}{n^2}$ converges.

(c) State Leibnitz test for alternating series. Also, give an example of each (with justification).

(i) a convergent series $\sum_{n=0}^{\infty} a_n$ for which $\sum_{n=0}^{\infty} a_n^2$ diverges

(ii) a divergent series $\sum_{n=0}^{\infty} a_n$ for which $\sum_{n=0}^{\infty} a_n^2$ converges.

(d) Determine which of the following series converge:

$$(i) \quad \sum_{n=2}^{\infty} \frac{1}{\sqrt{n}\,\log n}$$

$$(ii) \quad \sum_{n=0}^{\infty} \frac{(-1)^n\, n!}{2^n}$$

(23)

This question paper contains 2 printed pages. 11/12/17

Your Roll No. ....

Sl. No. of Ques. Paper: 5708 **H**

Unique Paper Code : 235104

Name of Paper : Algebra – I (MAHT-103)

Name of Course : B.Sc. (Hons.) Mathematics

Semester : I

Duration : 3 hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All six questions are compulsory.

Attempt any two parts from each question.

1. a) Compute the product using the polar representation of complex number (6)

$$(\tfrac{1}{2} - i\tfrac{\sqrt{3}}{2})(-3 + 3i)(2\sqrt{3} + 2i).$$

b) Find $|z|$ and $\arg z$ for (6)

$$z = (1 + \sqrt{3}i)^n + (1 - \sqrt{3}i)^n.$$

c) Solve the equation $z^{10} - z^5 + 1 = 0.$ (6)

2. a) For $a,b \in \mathbb{Z} - \{0\}$, define $a \sim b$ if and only if $ab>0$. (6)

i. Prove that $\sim$ defines an equivalence relation on $\mathbb{Z} - \{0\}$.

ii. What is an equivalence class of 5? What is an equivalence class of (-5)?

b) Given natural numbers a and b show that there are unique non negative integers q and r with $0 \le r < b$ s.t. $a = bq + r$. (6)

(c) Let $A = \{x \in \mathbb{R} \mid x \neq 2$ and $B = \{x \in \mathbb{R} \mid x \neq 1\}$ (6)

Define $f : A \to B$ and $g : B \to A$ by

$$f(x) = \frac{x}{x-2} \quad \text{and} \quad g(x) = \frac{2x}{x-1}$$

i. Find $f \circ g(x)$.

ii. (ii) Are f and g inverses? Explain

3. a) For every positive integer n prove that a set with exactly n-elements has exactly 2^n subsets (counting the empty set and entire set). (6)

P. T. O.

b) Show that if A and B are countable sets then so is A x B. (6)

c) If $a \equiv b \pmod{n}$ prove that gcd (a,n) = gcd(b,n). (6)

4. a) Let $(6\frac{1}{2})$

$$A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \qquad b = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$$

Determine if b is a linear combination of the vectors formed from the columns of the matrix A.

b) Find the parametric solution of the linear equation whose augmented matrix is $(6\frac{1}{2})$

$$\begin{bmatrix} 1 & -3 & -5 & 0 \\ 0 & 1 & 1 & 3 \end{bmatrix}.$$

c) Determine if the columns of the following matrix form a linearly independent set. Justify your answer. $(6\frac{1}{2})$

$$A = \begin{bmatrix} 1 & 4 & -3 & 0 \\ -2 & -7 & 5 & 1 \\ -4 & -5 & 7 & 5 \end{bmatrix}$$

5.

a) Find a basis for the null space of the Matrix $\begin{bmatrix} -4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix}$. $(6\frac{1}{2})$

b) Let $v_1 = \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, x = \begin{bmatrix} 3 \\ 12 \\ 7 \end{bmatrix}$. Is $B = \{v_1, v_2\}$ a basis of $H = \text{Span}\{v_1, v_2\}$. $(6\frac{1}{2})$

Determine if x is in H and if it is, find the coordinate vector of x relative to B.

c) Find the inverse of the Matrix $A = \begin{bmatrix} 1 & -2 & -1 \\ -1 & 5 & 6 \\ 5 & -4 & 5 \end{bmatrix}$, if it exists. $(6\frac{1}{2})$

6

a) Let $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$. Find a basis for the corresponding eigenspace for the eigenvalue 2. $(6\frac{1}{2})$

b) Find the characteristic equation and eigenvalues of $A = \begin{bmatrix} 1 & -6 \\ 4 & 5 \end{bmatrix}$ and discuss it. $(6\frac{1}{2})$

c) Is 5 an eigenvalue of $A = \begin{bmatrix} 6 & -3 & 1 \\ 3 & 0 & 5 \\ 2 & 2 & 6 \end{bmatrix}$? $(6\frac{1}{2})$

(24) 01/12/17

(This Question Paper contains 3 printed pages)

Roll N0...............................

Sr. No. of Question Paper: 8852

Unique Paper code: 222181

Name of the Course: B.Sc. (Hons) Mathematics / B.Sc. Math. Sciences (Credit course)

Name of the Paper: Physics-I

Semester: ~~I Semester~~

Duration: 3 hour

Max marks: 75

(Attempt Five questions in total. Use of nonprogrammable calculator is allowed)

Q1. (a) State and prove Gauss divergence theorem of vector calculus. (2,6)

(b) What are polar vectors? Give two examples of polar vectors. (2,1)

(c) Given $\vec{A}$ is a vector quantity. Prove the following vector identities:

(1) $\vec{\nabla}.(\vec{\nabla} \times \vec{A}) = 0$

(2) $\vec{A} \times (\vec{B} \times \vec{C}) + \vec{B} \times (\vec{C} \times \vec{A}) + \vec{C} \times (\vec{A} \times \vec{B}) = 0$ (2,2)

Q2. (a) What are conservative forces? Give two examples of conservative forces. (2,2)

(b) State and prove work energy theorem. (2,5)

(c) Two particles of masses 40g and 20g are moving opposite to each other with speed 2m/s and 4m/s respectively, on a straight line. They collide elastically in one dimension. Find the kinetic energy of each of them after collision.

(4)

Q3. (a) Prove the law of conservation of angular momentum? Give two suitable examples where law holds good.

(3.3)

(b) A solid cylinder of mass 100g and radius 4cm is rotating about its own axis. It completes 20 revolutions in 2 minute, calculate its

(1) Moment of inertia

(2) Angular momentum, and

(3) Rotational kinetic energy

(2,2,2)

(c) What is non-inertial frame of reference and pseudo forces?

(2, 1)

Q4. (a) Define damping factor in oscillations? Give one example of damped oscillation.

(1, 1)

(b) Write the differential equation for a damped harmonic oscillator. Solve the differential equation for its displacement for under-damped condition only.

(3, 5)

(c) What are Lissajous figures? Compute (graphically or analytically), the result of two simple harmonic vibrations of same amplitudes and at right angle to each other when their frequencies are in the ratio of 1:1 for phase difference of (a) 0 (b) $\frac{\pi}{4}$ and (c) $\frac{\pi}{2}$.

(2,3)

Q5. (a) Describe with necessary theory, the Young's double slit method of determining wavelength of monochromatic light.

(8)

(b) Two glass plates enclose a wedge shaped air film, touching at one edge and separated by a wire of 0.1mm diameter at a distance 10cm from the edge. Film is illuminated with monochromatic light of wave length 5000Å. Calculate the fringe width.

(4)

(c) What are the essential conditions for obtaining sustained interference?

(3)

Q6.(a) Differentiate between Fresnel and Fraunhoffer diffraction.

(4)

(b) Obtain analytically, the intensity relation for principal maxima of diffraction pattern due a plane transmission grating.

(8)

(c) A parallel beam of monochromatic light of wave length 5893Å is incident perpendicularly on a single slit of width 0.1mm. Calculate the angular width of central maximum.

(3)

Q7. (a) Describe construction and working of Nicol prism to obtain plane polarized light.

(4, 4)

(b) Differentiate between quarter and half wave plates.

(4)

(c) How one can distinguish ~~between~~ between circularly and elliptically polarized light experimentally?

(3)

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(1)

[This question paper contains 4 printed pages.]

4/12/17

Your Roll No..............

Sr. No. of Question Paper : 6621 **HC**

Unique Paper Code : 32351101

Name of the Paper : Calculus

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the sections are compulsory.
3. **All** questions carry equal marks.
4. Use of non-programmable scientific calculator is allowed.

Section – I

Attempt any **four** *questions from* **Section I.**

1. If $y = \cos(m \sin^{-1} x)$ then show that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0.$$

2. Sketch the graph of $f(x)=\dfrac{x^2-x-2}{x-3}$ by finding intervals of increase and decrease, critical points, points of relative maxima and minima, concavity of the graph and inflection points.

3. Find the horizontal asymptote to the graph of the function

$$f(x)=x^5\left[\sin\frac{1}{x}-\frac{1}{x}+\frac{1}{6x^3}\right]$$

4. A carpenter wants to make an open-topped box out of a rectangular sheet of tin 24 inches wide and 45 inches long. The carpenter plans to cut congruent squares out of each corner of the sheet and then bend and solder the edges of the sheet upward to form the sides of the box. For what dimension does the box have the greatest possible volume?

5. Sketch the graph of the curve in polar coordinates $r = 1 - 2\sin\theta$.

Section – II

Attempt any **four** *questions from* **Section-II.**

6. Find the reduction formula for $\int \sin^m x \cos^n x\,dx$ where m, n being positive integers and hence evaluate $\int_0^{\frac{\pi}{2}} \sin^5 x \cos^6 x\,dx$.

7. Find the volume of the solid generated when the region enclosed by the curve $y = \sqrt{x}$, $y = 3$, and $x = 0$ is revolved about the y-axis.

8. Find the volume of the solid generated when the region enclosed by the curve $y = x^2 + 1$, $y = x$, $x = 0$ over the interval [0, 3] revolved about the x-axis.

9. Find the arc length of the parametric curve $x = \sin 2t$, $y = \cos 2t$ for $0 \le t \le \frac{\pi}{2}$.

10. Find the area of the surface generated by revolving the curve $y = x^3$, $0 \le x \le 1$, about the x-axis.

Section – III

Attempt any **three** *questions from* **Section-III.**

11. Find the equation of a ellipse with foci at (2, 3) and (2, 5) and vertices (2, 2) and (2, 6).

12. Find the foci and equation of the hyperbola with vertices $(0, \pm 2)$ and asymptote $y = \pm 2x$.

13. Describe the graph of the equation $16x^2 - 9y^2 - 64x - 54y + 1 = 0$.

P.T.O.

14. Trace the conic $9x^2 - 24xy + 16y^2 - 80x - 60y + 100 = 0$ by rotating the coordinate axes to remove the xy term.

Section – IV

Attempt any **four** *questions from* **Section-IV**.

15. A particle moves with position function

$$\vec{F}(t) = (t \ln t)\hat{i} + (\sin t)\hat{j} + e^{-t}\hat{k}.$$

Find the velocity, speed and acceleration of the particle.

16. A shell is fired with muzzle speed 150 m/s and angle of elevation 45° from a position that is 10 m above the ground level. Where does the projectile hit the ground and with what speed?

17. Find the tangential and normal components of acceleration of an object that moves with position vector $R(t) = (t^3, t^2, t)$.

18. An object moves along the curve

$r = \sin\theta$ and $\theta = 2t$

Find its velocity and acceleration in terms of unit polar vectors u_r and u_θ.

19. Find the curvature and radius of curvature of the twisted cubic for a curve

$r(t) = \{t, t^2, t^3)$ at a general point and at (0, 0, 0).

2

[This question paper contains 6 printed pages.]

15/7/17

Your Roll No...............

Sr. No. of Question Paper : 6622 **HC**

Unique Paper Code : 32351102

Name of the Paper : Algebra

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All **Six** questions are compulsory.
3. Do any **two** parts from each question.

1. (a) Find all complex numbers z, such that $|z| = 1$ and

$$\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1.$$ (6)

(b) Find the fourth roots of unity and represent them in the complex plane. Show that they form a square inscribed in the unit circle. (6)

P.T.O.

(c) Solve the equation

$$z^6 + iz^3 + i - 1 = 0. \qquad (6)$$

2. (a) For $a, b \in Z$, define $a \sim b$ if and only if $3a + b$ is a multiple of 4.

(i) Prove that $\sim$ defines an equivalence relation.

(ii) Find the equivalence class of 0 and 2. (6)

(b) Let $\sim$ denote an equivalence relation on a set A and $a \in A$. Prove that for any $x \in A$, $x \sim a$ if and only if $\bar{x} = \bar{a}$, where $\bar{x}$ denotes the equivalence class of x. (6)

(c) Show that Z and 3Z have the same cardinality. (6)

3. (a) Using Euclidean algorithm find g.c.d [1004, –24) and express it as an integral linear combination of the given integers. (6)

(b) Find $(1017)^{12}$ (mod 7). (6)

(c) Using Principle of Mathematical Induction, prove that for every positive integer n, $n^3 + 2n$ is divisible by 3. (6)

4. (a) Find the general solution to the linear system whose augmented matrix is

$$A = \begin{bmatrix} 1 & 1 & 0 & 2 & -3 & 2 \\ 1 & 1 & 1 & 2 & -3 & 3 \\ 2 & 1 & 0 & 2 & -3 & 4 \\ 4 & 3 & 1 & 1 & -9 & 9 \end{bmatrix}$$

by row reducing the matrix to Echelon Form. Encircle the leading entries, list the basic variables and free variables. Write the general solution in Parametric Vector Form. (6½)

(b) Define Linearly Dependent Set.

Let $v_1 = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$, $v_2 = \begin{bmatrix} 3 \\ -5 \\ 10 \end{bmatrix}$, $v_3 = \begin{bmatrix} -1 \\ 5 \\ h \end{bmatrix}$ for what value(s) of h, the set $\{v_1, v_2, v_3\}$ is

(i) Linearly Independent

(ii) Linearly Dependent. (6½)

(c) Let $v_1 = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix}$, $v_3 = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$

Do the vectors v_1, v_2, v_3 span $\mathcal{R}^3$? Justify. Hence or otherwise express $v = \begin{bmatrix} 8 \\ -4 \\ 2 \end{bmatrix}$ as linear combination of v_1, v_2, v_3. (6½)

5. (a) Boron sulphide reacts violently with water to form boric acid and hydrogen sulphide gas. The unbalanced equation is

$$B_2S_3 + H_2O \rightarrow H_3BO_3 + H_2S$$

Balance the chemical equation using the vector equation approach. (6½)

(b) Let T: $\mathcal{R}^2 \rightarrow \mathcal{R}^2$ be a linear transformation such that T first rotates through $\frac{\pi}{2}$-radians in the anti-clockwise direction and then reflects through the line $x_1 = x_2$. Find the Standard matrix of T. (6½)

(c) Let T: $\mathcal{R}^2 \to \mathcal{R}^2$ be defined as $T(x_1, x_2) = (x_2 - x_1, 2x_2 + x_1)$ be a linear transformation. Prove that T is invertible and find a rule for T^{-1}. (6½)

6. (a) Let

$$A = \begin{bmatrix} 1 & -1 & 5 \\ 2 & 0 & 7 \\ -3 & -5 & -3 \end{bmatrix} \text{ and } u = \begin{bmatrix} -7 \\ 3 \\ 2 \end{bmatrix}$$

Is u in Nul A ? Is u in Col A ? Justify each answer. (6½)

(b) (i) Suppose a 4 × 7 matrix A has three pivot columns. Is Col A = $\mathcal{R}^3$? What is the dimension of Nul A ? Explain your answer.

(ii) Consider the basis $B = \left\{ \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix} \right\}$ for $\mathcal{R}^2$. If $[x]_B = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$, find the vector x. (3½,3)

(c) For the matrix given below, find the characteristic equation and the eigen values with their multiplicities. Also, find a basis for the eigenspace corresponding to any one of the eigenvalues.

$$A = \begin{bmatrix} 5 & 8 & 0 & 1 \\ 0 & -4 & 7 & -5 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ (6½)